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# Measurement of the Shear Strength of an Expansive Soil by Combining a Filter Paper Method and Direct Shear Tests

**ABSTRACT:** The measurement of the shear strength of unsaturated soils in terms of two independent stress state variables is usually difficult, expensive, and time-consuming. This paper presents a proposal to combine a filter paper method and a conventional direct shear test to obtain this measurement. The feasibility of this approach is illustrated through tests on an expansive soil. First, the filter paper method is used to establish the soil-water characteristic curve of the soil, and a series of conventional direct shear tests is subsequently conducted to measure the shear strength of the soil. The matric suction of the soil at failure is estimated from the soil-water characteristic curve based on the water content of the soil tested. The test results show that the failure envelopes of the expansive soil are nonlinear on the shear strength versus the matric suction plane for different net normal stresses. The unsaturated shear strength parameter $\varphi^b$ is equal to the effective friction angle $\varphi'$ when the soil is close to being saturated, and $\varphi^b$ decreases as the soil becomes drier. The combined method proposed in this paper may be a practical technique for analyzing the shear strength of unsaturated soils in terms of two independent stress state variables because it can be conducted in most geotechnical laboratories.

**KEYWORDS:** conventional direct shear test, filter paper method, shear strength, soil water characteristic curve, unsaturated expansive soil

## Introduction

The shear strength of a soil is important in many geotechnical problems, such as bearing capacity, lateral earth pressure, and slope stability (Fredlund and Rahardjo 1993), which is usually determined by the Mohr-Coulomb model. For a saturated soil, the effective stress variable of $\sigma_n - u_w$ is commonly used with the Mohr-Coulomb theory (Terzaghi 1936). For an unsaturated soil, the theory was extended by Fredlund et al. (1978) to include two independent stress state variables (i.e., the net normal stress ($\sigma - u_a$) and the matric suction ($u_a - u_w$)), and the extended shear strength equations have been verified to be reasonably accurate (Gan and Fredlund 1996; Oloo and Fredlund 1996; Gui and Yu 2008).

The shear strength of an unsaturated soil is usually measured in a laboratory using either a modified triaxial or direct shear apparatus in which the matric suction is controlled by applying the axis-translation or osmotic technique (Ng et al. 2007). Generally, the modified apparatus is sophisticated and expensive, and the shear strength test using this apparatus requires experience and special training and is time-consuming (Fredlund et al. 1996; Khalili and Khabbaz 1998). These disadvantages limit the wide application of the modified shear strength test techniques in many geotechnical engineering laboratories (at least currently in China), although the principle of the techniques is reasonable. Therefore, many studies on the shear strength of unsaturated soils still use conventional triaxial or direct shear tests without controlling for or measuring soil suction. Usually, these tests are regarded as "total stress" type tests, and the apparent shear strength parameters $c$ and $\phi$ are used to describe the shear strength. Fredlund and Rahardjo (1993) noted that the total stress approach can be applied in the field in cases in which the strength measured in the laboratory is assumed to be relevant to the drainage conditions being simulated in the field.

Several experiments have indicated that the apparent shear strength parameters of an unsaturated soil are closely related to the water content of the soil (Oloo and Fredlund 1996; Miao et al. 1999; Matsuoshi and Matsukura 2006; Ling and Yin 2007). The relationship between suction and the water content of soil is known as the soil-water characteristic curve (SWCC). Based on the SWCC, the suction of the soil tested can be indirectly obtained from the water content of the soil. Assuming that the pore air pressure is equal to the atmospheric pressure (i.e., 101 kPa), the shear strength of an unsaturated soil measured using conventional shear tests can subsequently be interpreted in terms of the independent net normal stress and matric suction stress state variables. Although the suction value obtained from the SWCC is approximate, it is suitable for the analysis of most practical problems (Fredlund et al. 1995). The prerequisite for this indirect suction measurement is to establish the SWCC for the soil. There are many methods for establishing the SWCC for unsaturated soils, among which the filter paper method is relatively easy.

In this study, the combination of a filter paper method and a conventional direct shear test was proposed to determine the shear strength parameters of an unsaturated soil. To illustrate the application of this new method, a series of laboratory tests were

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performed on identically compacted expansive soil specimens that were prepared by compacting soil-water mixtures of equal initial water content to produce a predetermined dry density (Fredlund and Rahardjo 1993). Because the filter paper method has seldom been used in China (Jiang et al. 2000; Wang et al. 2003), a Chinese domestic filter paper with the brand name Double-Circle No. 402 was first calibrated and was used to determine the SWCC of the soil tested. The relationship between the apparent shear strength parameters and the water content of the soil was obtained from ten groups of conventional direct shear tests. Finally, the shear strength parameters of the soil in terms of the extended Mohr-Coulomb failure criteria were determined based on the SWCC and the shear strength of the soil tested.

## The Filter Paper Method

The filter paper method is an indirect way to conduct the matric or total suction measurement, which is based on the assumption that the moisture of the testing filter paper will come into equilibrium with that of an unsaturated soil, and the soil suction can subsequently be obtained from the calibration curve of the testing filter paper. Gardner (1937) was probably the first researcher to use the filter paper method to measure soil suction (Marinho and Oliveira, 2006). The filter paper method has also been used by Al-Khafaf and Hanks (1974), van der Raat et al. (1987), Houston et al. (1994), Leong and Rahardjo (2002), Bulut and Wray (2005), Patrick et al. (2007), Feuerharmel et al. (2006), Bulut and Leong (2008), Tripathy and Subba Rao (2009), and Nam et al. (2009). Bulut and Wray (2005) reported that the filter paper method can be a reliable soil suction measurement technique if the basic principles of the method are well understood, and a strict laboratory protocol is carefully followed.

As a porous material, filter paper has the ability to retain water like a soil, and the suction of the filter paper will be the same as that of the soil at equilibrium in a closed container (Bulut and Wary, 2005). Because equilibrium can be reached by either liquid or vapor moisture exchange between the soil and the filter paper, there are two ways for a filter paper to acquire water. When the filter paper directly contacts the soil, it is assumed that the filter paper and the soil can freely exchange water and solutes in a liquid phase. This direct contact method is used to measure the matric suction of the soil, whereas if the filter paper is suspended above the soil (i.e., the non-contact method), both the matric and osmotic suction of the soil will contribute to the vapor pressure of the soil water and increase the relative humidity in the air. Therefore, the equilibrated water content in the filter paper, which increases as a result of vapor flow, reflects the total suction of the soil.

The application of the filter paper method in this study consists of three main steps: (1) the calibration of a Chinese domestic filter paper with the brand name Double-Circle No. 402, (2) the measurement of an expansive soil suction using the filter paper method, and (3) the establishment of the SWCC of the soil. In the first step, the relationship between the suction (i.e., the total suction $h_t$ or matric suction $h_m$) and the water content of the filter paper, $w_{fp}$, is established. In the second step, the filter paper and the soil specimen are sealed in glass jars using either the direct contact or the non-contact method for equilibrium, and the equilibrated water content of the filter paper is measured. On the basis of the suction calibration curves of the filter paper, the soil suction is indirectly deduced from the measured water content of the filter paper. Subsequently, the SWCC of the soil is obtained from the soil specimens with different water contents.

## The Calibration of the Double-Circle No. 402 Filter Paper

Because the filter paper method is an indirect technique to measure soil suction, its measurement accuracy is highly dependent on the suction calibration curve of the testing filter paper. Considerable numbers of measurements have been conducted on the two typical filter papers, Whatman No. 42 and Schlecher & Schuell No. 589. The details of the laboratory protocols and procedures for the application are suggested by the ASTM Standard Test Method for Measurement of Soil Potential (Suction) Using Filter Paper (ASTM D5298). In this study, a Chinese domestic filter paper with the brand name Double-Circle No. 402 was calibrated and used to measure the suction of the soil tested.

The configuration of the suction calibration test on the Double-Circle No. 402 filter paper is shown in Fig. 1. The total suction calibration test was conducted in a wetting process using NaCl solutions prepared from 0.001 to 2.5 molality (i.e., the number of moles of NaCl in mass in 1000 ml of distilled water), which corresponded to total suction values ranging from 5 to 12,556 kPa at 25°C (Lang, 1967). The filter paper and the NaCl solution were sealed in a glass jar without contact for moisture equilibration

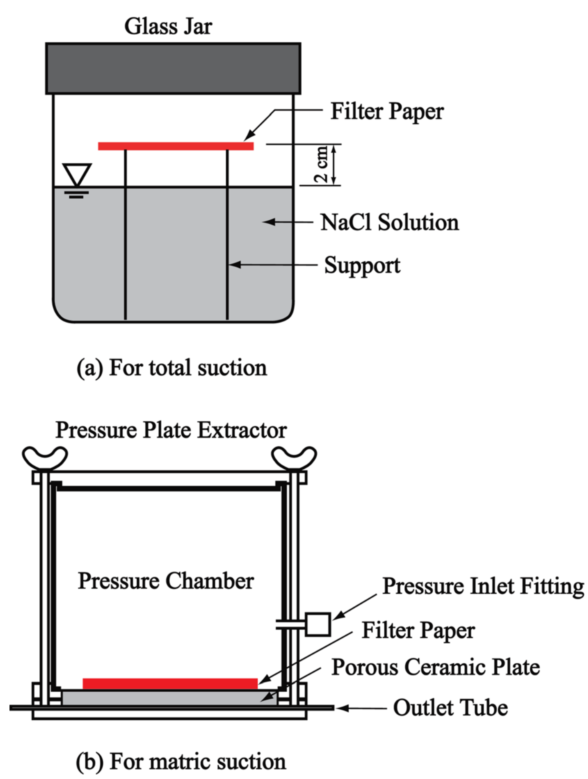


FIG. 1—*Configuration of the total and matric suction calibration test on the filter paper.*

(Fig. 1(*a*)). To minimize the temperature fluctuation, the glass jars were placed in a water bath with a constant temperature of 25°C. After 10 days of equilibration (Sibley and Williams 1990), the wet filter papers were weighed to the nearest 0.0001 g. The filter papers were then dried in an oven for 24 h at 105°C and weighed again to obtain the equilibrium water content of the filter papers. The matric suction calibration test was conducted with a drying process using a pressure plate extractor with a 2-bar porous ceramic plate, which permits the operation of the extractor at any pressure between 0 and 200 kPa. Both the porous ceramic plate and the filter paper were saturated prior to the calibration. The saturated filter papers were placed in the pressure extractor in close contact with the porous ceramic plate (Fig. 1(*b*)). Next, the pressure chamber was tightened, and the air pressure in the extractor was set at a desired value. After 10 days of equilibration between the filter papers and the porous ceramic plate, the water content of the filter paper was measured using the same procedures used in the total suction calibration test. Additional details of the laboratory protocols and procedures for the filter paper method can be found in the ASTM Standard Test Method for Measurement of Soil Potential (Suction) Using Filter Paper (ASTM D5298).

The calibration curves for the Double-Circle No. 402 filter paper are depicted in Fig. 2. Both the total suction and matric suction calibration curves can be described by a bi-linear regression curve that represents the different sensitivities of the filter paper response in the higher and lower suction ranges (Leong et al. 2002; Bulut and Wray, 2005). For the total suction calibration curve, the water content at the breakpoint is approximately 41 %, and the bi-linear regression curve is expressed as follows:

$$\left.\begin{array}{ll}\log(h_t) = 5.257 - 0.070w_{fp}, & w_{fp} \leq 41\,\% \\ \log(h_t) = 51.321 - 1.194w_{fp}, & w_{fp} > 41\,\%\end{array}\right\} \quad (1a)$$

For the matric suction calibration curve, the water content at the breakpoint is approximately 47 %, and the bi-linear regression curve can be expressed as follows (Wang et al. 2003):

$$\left.\begin{array}{ll}\log(h_m) = 5.493 - 0.076w_{fp}, & w_{fp} \leq 47\,\% \\ \log(h_m) = 2.470 - 0.012w_{fp}, & w_{fp} > 47\,\%\end{array}\right\}, \quad (1b)$$

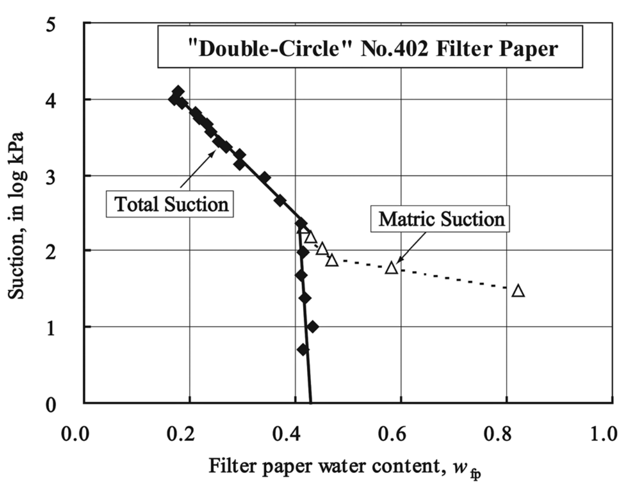


FIG. 2—*Calibration curve of the Double-Circle No. 402 filter paper.*

where:

$\log(h_t)$ and $\log(h_m)$ are the logarithm of the total and matric suction (kPa), respectively, and

$w_{fp}$ is the calibrated water content of the filter paper.

## Nanyang Expansive Soil

The expansive soil used in the study was taken from a construction field of the South-to-North Water Transfer Project in Nanyang, China. The physical properties of the soil, as determined per the Chinese soil testing standard SL237-1999, are listed in Table 1 in which the free swelling ratio (FSR) is defined as the ratio of the equilibrium volume increment in a 50-ml graduated cylinder filled with 35 ml NaCl solution to the initial testing volume of the soil, oven-dried and passed through a 0.5-mm sieve. The soil has a FSR of 82 % and is classified as a medium expansive soil. The gradation of the soil is shown in Fig. 3. The fractions of sand (2000 $\mu$m $\geq$ d >75 $\mu$m), silt (75 $\mu$m $\geq$ d >5 $\mu$m) and clay (5 $\mu$m $\geq$ d) are 15 %, 62 %, and 23 %, respectively. X-ray diffraction analysis shows that the clay mineral components of illite, montmorillonite, and kaolinite in the soil are 33 %, 19 %, and 8 %, respectively.

## Specimen Preparation and Suction Measurement

The SWCC for the soil was established using the calibrated "Double-Circle" No. 402 filter paper. The soil was screened through a 2-mm sieve, oven-dried and mixed with the required amount of distilled water for the optimum water content of 20.4 %. When the moisture equilibrium of the soil was reached in a sealed container for approximately 24 h, eight identical specimens (6.18 cm in diameter and 4 cm in height) with a dry density of 1.6 g/cm$^3$ were prepared by static compaction. Each soil specimen was compacted in two layers such that the filter papers could be easily placed within the specimen for the direct contact measurement.

To establish the SWCC of the soil, the water content of the eight identical soil specimens was changed by wetting or drying. As presented in Table 2, one specimen was dried in the air, one was dried in a low-temperature oven, one was sealed in a glass jar to maintain its water content, three were wetted in a water bath, and the remaining two specimens were saturated in water under a vacuum. After reaching the desired water content, the specimens were sealed in glass jars to allow moisture equilibration for one week.

Figure 4 shows the configuration of the suction measurement test. When equilibrium was attained, one specimen was taken out of the steel ring and divided into two parts along its compaction

TABLE 1—*Physical properties of the Nanyang expansive soil.*

| Physical Properties | Values |
|---|---|
| Liquid limit (%) | 50.1 |
| Plasticity limit (%) | 26.8 |
| Free swelling ratio (FSR) (%) | 82.0 |
| Special gravity | 2.45 |
| Maximum dry density (g/cm$^3$) | 1.76 |
| Optimum moisture content (%) | 20.4 |

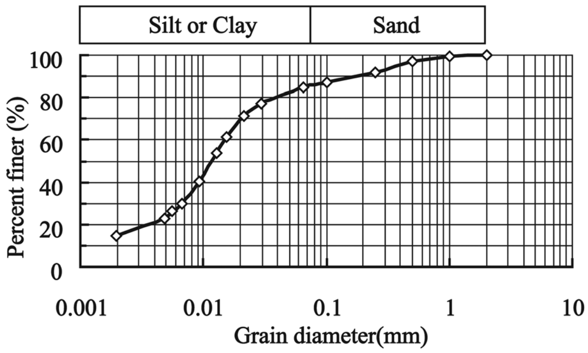


FIG. 3—*Grain size distribution of the soil tested.*

layer. A piece of the Double-Circle No. 402 filter paper with a diameter of 50 mm, sandwiched between two pieces of the protective filter paper with a diameter of 60 mm, was carefully placed between the two parts of the specimen for the measurement of the matric suction. The two parts of the specimen, with the filter papers having been inserted between them, were joined with a piece of adhesive tape to form a single specimen, which was subsequently placed in a clean, dry glass jar. Two pieces of the dry filter paper with diameters of 50 mm were placed on a ring support over the soil specimen for the total suction measurement. Finally, the glass jar was sealed and placed in a water bath to allow moisture equilibration for ten days during which the temperature in the bath was maintained at 25°C.

Upon equilibration, both the direct and non-contact wet filter papers were weighed to the nearest 0.0001 g. The wet soil, taken from the mid-height plane of the specimen, was also weighed. The wet filter papers and the wet soil were subsequently oven dried to a constant weight at 105°C and weighed to obtain their respective water contents.

## *Test Results*

The measured water content of the soil specimens and filter papers and the estimated soil suction values are listed in Table 2. The total suction was estimated from the water content of the non-contact filter paper using Eq 1(*a*), and the matric suction was estimated from the water content of the direct contact filter papers using Eq 1(*b*); the osmotic suction is the difference between the total and matric suctions.

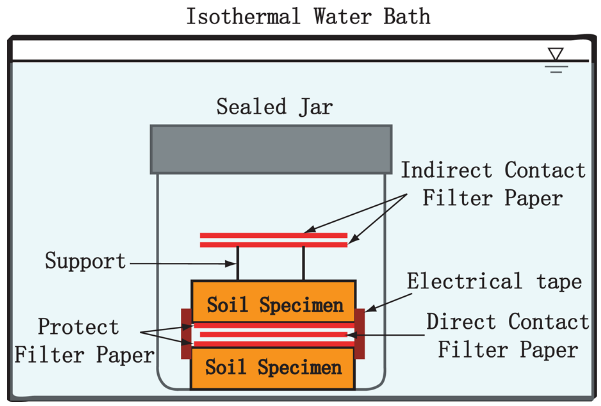


FIG. 4—*Test configuration of soil suction measurement.*

Figure 5 shows the SWCCs of the soil tested by plotting the estimated suction values in a logarithmic scale against the water content of the soil. The values for matric suction, $h_m$, and gravimetric water content, $w_s$, are best fitted by the following equation, as proposed by Fredlund and Xing (1994):

$$w_s = 29.8 \times \left[1 - \frac{\ln\left(1 + \frac{h_m}{1000}\right)}{\ln\left(1 + \frac{10^6}{1000}\right)}\right] \left[\frac{1}{\left\{\ln\left[\exp(1) + \left(\frac{h_m}{107.4}\right)^{3.4}\right]\right\}^{0.2}}\right] \quad (2)$$

Figure 5 demonstrates that both the matric suction and the total suction of the soil tested increase greatly, and their relevant SWCCs tend to converge as the water content of the soil decreases, i.e., the change in total suction is essentially equivalent to that of the matric suction when the soil is relatively dry. The osmotic suction, which is mainly attributable to the dissolved salts contained in the soil water, is nearly constant when the soil water is greater than 21 %.

Figure 6 shows typical matric, osmotic, and total suction values for a remolded glacial till (Krahn and Fredlund, 1972) in which the total suction, the matric suction, and the osmotic suction were

TABLE 2—*Results of the suction measurement of the soil tested.*

| | Soil Specimen | | Non-contact Filter Paper | | Direct Contact Filter Paper | | |
|---|---|---|---|---|---|---|---|
| Set No. | Soil Moisture Control Path | Water Content of Soil Specimen, $w_s$ (%) | Water Content of Filter Paper, $w_{fp}$ (%) | Total Suction, $h_t$ (kPa) | Water Content of Filter Paper, $w_{fp}$ (%) | Matric Suction, $h_m$, (kPa) | Osmotic Suction, $h_o = h_t - h_m$ (kPa) |
| 1 | Oven dried | 16.5 | 24.99 | 3283 | 28.41 | 2157 | 1126 |
| 2 | Air dried | 18.1 | 28.50 | 1869 | 34.96 | 686 | 1183 |
| 3 | Maintained | 21.0 | 31.68 | 1123 | 39.70 | 299 | 824 |
| 4 | Wetted in the | 22.5 | 32.56 | 974 | 39.15 | 329 | 645 |
| 5 | water bath | 25.8 | 34.23 | 746 | 43.26 | 160 | 585 |
| 6 | | 26.5 | 34.16 | 754 | 50.98 | 72 | 682 |
| 7 | Saturated under | 29.7 | 35.33 | 625 | 54.56 | 65 | 560 |
| 8 | vacuum | 29.9 | 35.26 | 632 | 61.97 | 53 | 578 |

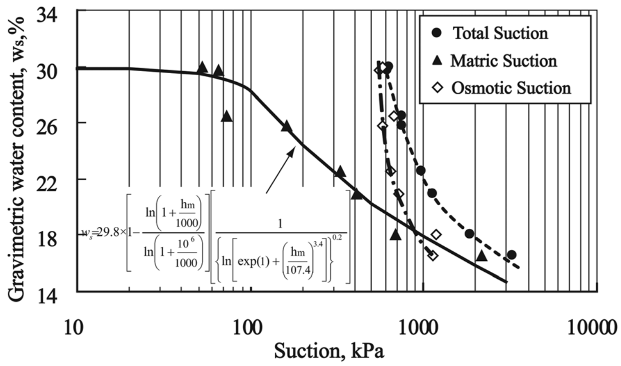


FIG. 5—*Total, matric and osmotic suctions, in kPa units, versus the water content for the soil tested.*

measured with a psychrometer, a pressure plate, and the squeezing technique, respectively. A comparison of Fig. 5 with Fig. 6 reveals that the variation regularities of the three SWCCs established by the filter paper method proposed in this paper are similar to the variations determined by direct suction measurements, verifying that the filter paper method proposed in this paper is appropriate for establishing the SWCC of an unsaturated soil.

## Direct Shear Tests

A series of conventional consolidated undrained direct shear tests were performed according to the Chinese soil-testing standard SL237-1999. The shear strength parameters in terms of the two independent stress state variables were determined.

The direct shear tests were conducted on 10 specimens with different water content values. The preparation of the soil specimens was similar to that used in the suction measurement tests. The soil was screened through a 2-mm sieve, oven-dried and mixed with the required amount of distilled water for the optimum water content of 20.4 %. After 24 h of equilibration in a sealed container, the soil was statically compacted into 40 representative soil specimens of 6.18 cm in diameter and 2 cm in height with a dry density of 1.6 g/cm$^3$. These 40 soil specimens were divided into 10 groups (four specimens per group), labeled A to J. Specimens of different groups were then altered to different initial water contents by means of oven drying, air drying, wetting, or air

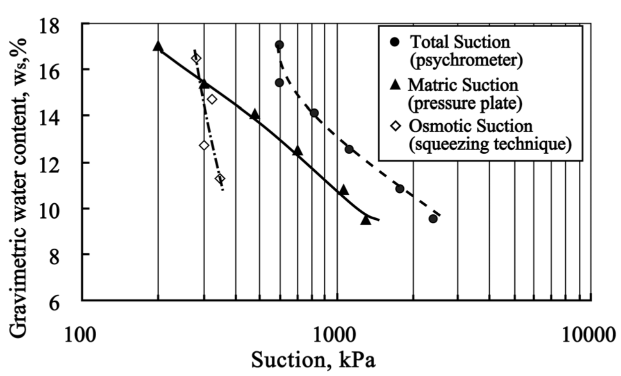


FIG. 6—*Total, matric, and osmotic suctions for glacial till (from Krahn and Fredlund, 1972).*

exhaust, as shown in Table 3. When reaching the desired water content, which was determined by the mass change of the soil specimen, each group of specimens was sealed in a glass container for moisture equilibration for one week. When equilibrium was achieved, conventional consolidated undrained direct shear tests were conducted on the 10 groups of specimens in sequence. The specimens in each group were sheared at a rate of 1 mm/min with applied normal stresses of 100, 200, 300, and 400 kPa. At the end of each test, the shear box was quickly dismantled, and the water content of the soil in the failure zone was measured.

### *The Results of the Direct Shear Tests*

The results of the conventional direct shear tests on the soil are presented in Table 3 in which the mean water content of the four soil specimens at failure in each group is provided. The total and matric suctions of the soil specimens are estimated from the SWCCs, as shown in Fig. 5. The estimated matric suction of 58 kPa in the saturated specimens in Group J may be the result of experimental error, and it is assumed to be zero for the shear strength analysis in the following paragraphs.

The results of conventional direct shear tests can be analyzed using a total stress approach, as follows:

$$\tau_f = c + (\sigma_n)_f \tan\phi \quad (3)$$

where:

$\tau_f$ and $(\sigma_n)_f$ are the shear stress measured at failure and the total normal stress applied on the failure plane, respectively,

$c$ is the so-called apparent cohesion, and

$\phi$ is the apparent angle of internal friction.

The measured shear strengths of the compacted expansive soil are plotted against the applied normal stresses in Fig. 7. The variation of the measured shear strengths with the applied normal stresses (i.e., the shear strength envelope) is nearly linear for each water content value. From these shear strength envelopes, the apparent cohesions, *c,* and the apparent friction angles, $\phi$, associated with different water contents are presented in Table 3.

The variation in both the apparent cohesion and the apparent friction angle with the water content of the soil at failure is shown in Fig. 8. For the soil tested, the apparent cohesion in log kPa units decreases approximately linearly with an increase in water content. In accordance with the data in Table 3, the apparent friction angle decreases from 20° at a water content of 16.4 % to 10.4° at a water content of 22.8 %, whereas it remains at a constant value of approximately 10° at water contents ranging from 22.8 % to 29.3 %. The reason for the substantially wide apparent friction angle of the soil in a relatively dry condition is unclear and needs to be studied further. Matsuoshi and Matsukura (2006) reported that under excessively dry conditions, the cementation of fines and/or shrinkage of the soil mass may contribute to a greater frictional resistance of soil aggregates.

Because the SWCC of the soil was determined by the filter paper method in the present study, the results of the direct shear tests could also be analyzed in terms of the two independent stress state variables, i.e., the net normal stress $(\sigma - u_a)$ and the matric suction $(u_a - u_w)$, as proposed by Fredlund et al. (1978). The shear strength equation is written as

TABLE 3—*Results of conventional direct shear tests.*

| Group No. | Moisture Control Path | Mean Value of Water Content, $w_s$ (%) | Total Suction, $h_t$ (kPa) | Matric Suction, $h_m$ (kPa) | Applied Normal Stress, $\sigma_n$ (kPa) | Measured Shear Strength, $\tau$(kPa) | Apparent Cohesion, $c$ (kPa) | Apparent Friction Angle, $\phi$ (degrees) |
|---|---|---|---|---|---|---|---|---|
| A | Oven dried | 16.4 | 3300 | 1707 | 100 | 255 | 218.5 | 20.0 |
| | | | | | 200 | 287 | | |
| | | | | | 300 | 336 | | |
| | | | | | 400 | 360 | | |
| B | Air-dried | 18.0 | 1875 | 999 | 100 | 214 | 179.0 | 19.2 |
| | | | | | 200 | 250 | | |
| | | | | | 300 | 280 | | |
| | | | | | 400 | 320 | | |
| C | Maintained | 20.3 | 1225 | 497 | 100 | 172 | 138.0 | 14.9 |
| | | | | | 200 | 182 | | |
| | | | | | 300 | 215 | | |
| | | | | | 400 | 250 | | |
| D | Wetted in the water bath | 21.6 | 1050 | 356 | 100 | 145 | 115.0 | 13.8 |
| | | | | | 200 | 160 | | |
| | | | | | 300 | 180 | | |
| | | | | | 400 | 220 | | |
| E | | 22.4 | 960 | 298 | 100 | 118 | 95.5 | 11.4 |
| | | | | | 200 | 130 | | |
| | | | | | 300 | 160 | | |
| | | | | | 400 | 175 | | |
| F | | 22.8 | 925 | 275 | 100 | 114 | 97.5 | 10.4 |
| | | | | | 200 | 133 | | |
| | | | | | 300 | 160 | | |
| | | | | | 400 | 166 | | |
| G | | 23.1 | 905 | 258 | 100 | 110 | 94.0 | 9.8 |
| | | | | | 200 | 128 | | |
| | | | | | 300 | 150 | | |
| | | | | | 400 | 160 | | |
| H | | 26.1 | 750 | 156 | 100 | 90 | 76.0 | 10.0 |
| | | | | | 200 | 118 | | |
| | | | | | 300 | 126 | | |
| | | | | | 400 | 146 | | |
| I | | 27.3 | 700 | 124 | 100 | 90 | 72.0 | 9.8 |
| | | | | | 200 | 107 | | |
| | | | | | 300 | 120 | | |
| | | | | | 400 | 143 | | |
| J | Saturated under vacuum | 29.3 | 630 | 58 | 100 | 72 | 52.5 | 11.1 |
| | | | | | 200 | 90 | | |
| | | | | | 300 | 115 | | |
| | | | | | 400 | 129 | | |

$$\tau_f = c' + (\sigma_n - u_a)_f \tan\phi' + (u_a - u_w)_f \tan\phi^b \quad (4)$$

where:

$\tau_f$ is the shear stress on the failure plane at failure,

$c'$ is the effective cohesion,

$(\sigma_n - u_a)_f$ is the net normal stress on the failure plane at failure,

$\phi'$ is the angle of internal friction associated with the net normal stress state variable,

$(u_a - u_w)_f$ is the matric suction at failure, and

$\phi^b$ is the angle indicating the rate of increase in shear strength relative to matric suction.

In this paper, the matric suction at failure $(u_a - u_w)_f$, also designated $h_m$, was deduced from the SWCC of the soil tested through the water content of the soil specimen of the direct shear test at failure (Table 3). It is assumed that the pore-air pressure in the soil at failure is equal to the atmospheric pressure, and the net normal stress, $(\sigma_n - u_a)$, has an effectiveness that is similar to the total normal stress, $\sigma_n$. Equation 4 can then be revised as

$$\tau_f = c' + (\sigma_n)_f \tan\phi' + (h_m)_f \tan\phi^b \quad (5)$$

where the effective cohesion $c'$ and friction angle $\phi'$ are given from the Mohr-Coulomb failure envelope at the saturated

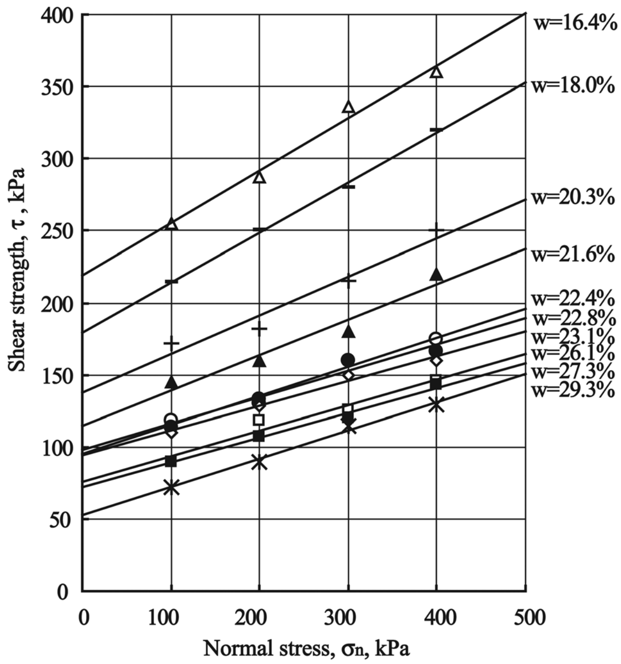


FIG. 7—*Shear strength versus the net normal stress at different initial water contents, $w_s$, for the soil tested.*

condition (i.e., $h_m = 0$), and the angle $\phi^b$ is obtained from the relationship between the shear strength and the matric suction.

For the soil tested in the study, the shear strength parameters of $c'$ and $\phi'$ are equal to 52.5 kPa and 11.1°, respectively. These values are obtained from the lowest strength envelope in Fig. 7, which corresponds to the tests of Group J in the saturated condition. The shear strengths of the soil against the matric suction are shown in Fig. 9(*a*). The slope of the shear strength versus the matric suction curve gives the value of tan$\phi^b$. Figure 9(*a*) illustrates that the failure envelopes show a significant nonlinear behavior, and the best-fit curves have a similar shape for different levels of normal stress. The failure envelopes for different levels of normal stress have the same initial slope angle equal to $\phi'$ (11.1°) at the lower matric suction, and the envelopes descend

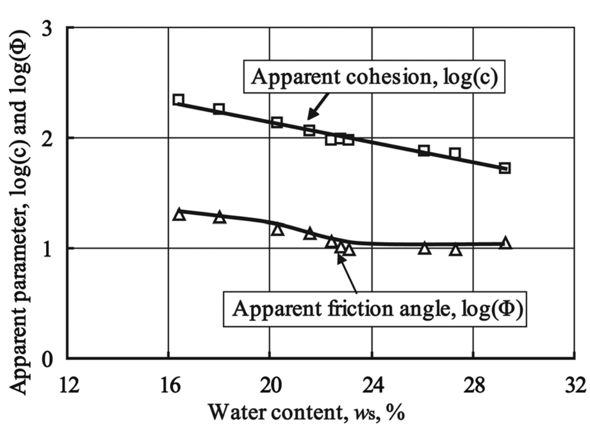


FIG. 8—*Apparent cohesion and friction angle versus the water content for the soil tested.*

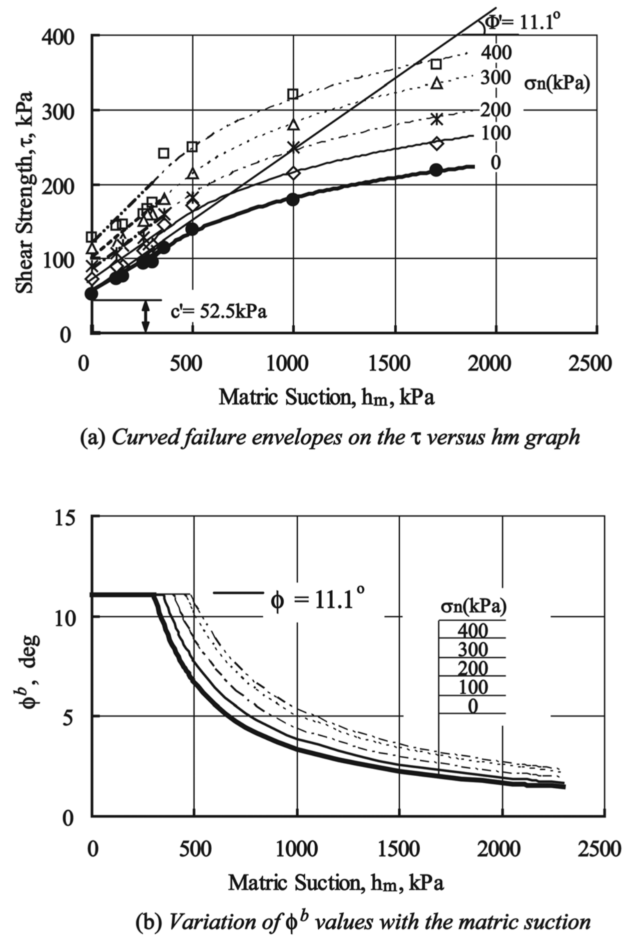


FIG. 9—*Nonlinearity of the failure envelopes with respect to the matric suction, $h_m$, for the soil tested.*

with the increase in the matric suction. The variation in the slope angle $\phi^b$ with respect to the matric suction for different levels of normal stress is shown in Fig. 9(*b*). The higher the net normal stress, the larger the matric suction, and the failure envelope begins to descend. A similar nonlinear behavior of the shear strength envelopes for unsaturated soils has also observed by several previous researchers (i.e., Fredlund et al. 1996, and Gan and Fredlund, 1996). Oloo and Fredlund (1996) proposed that the nonlinearity of the shear strength envelopes is the result of the diminishing contribution of matric suction to the shear strength, as the water content of the soil approaches its residual value.

## Concluding Remarks

Because of the simplicity, cost-effectiveness, and reduced time requirements, both the filter paper method and the conventional direct shear test have been widely used for testing unsaturated soils. In this study, the combination of the filter paper method and the conventional direct shear test is proposed to evaluate the shear strength characteristics of unsaturated soils. The filter paper method is used to establish the SWCC from which the suction of the direct shear test specimen is deduced through the water content of the shear test specimen. The feasibility of this combined method has been demonstrated by testing the Nanyang unsaturated expansive soil. The failure envelopes of the soil established by the combined method show a significant nonlinearity of the shear strength versus the matric suction plane. The values of $\phi^b$ are equal to the friction angle $\phi'$ (i.e., the friction angle measured under saturation condition) at lower matric suction, and they decrease as the matric suction increases to a certain value.

Although the proposed combined filter paper and conventional direct shear test method cannot fully replace more precise methods for analyzing the shear strengths of unsaturated soils, it is practical, cost-effective, and relatively easier to implement than other methods, and this method is capable of determining the approximate shear strength envelopes with respect to matric suction, as demonstrated in this study.

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## References

Al-Khafaf, S., and Hanks, R. J., 1974, "Evaluation of the Filter Paper Method for Estimating Soil Water Potential," *Soil Sci.*, Vol. 117, No. 4, pp. 194–199.

ASTM D5298-94 (1992), "Standard Test Method for Measurement of Soil Potential (Suction) Using Filter Paper," *Annual Book of ASTM Standards*, Vol. 04.08. ASTM International, West Conshohocken, PA, pp. 1312–1316.

Bulut, R., and Wray, W. K., 2005, "Free Energy of Water-Suction-In Filter Papers," *Geotech. Testing J.*, Vol. 28, No. 4, pp. 355–364.

Bulut, R., and Leong, E. C., 2008, "Indirect Measurement of Suction," *Geotech. Geol.Eng.*, Vol. 26, No. 6, pp. 633–644.

Feuerharmel, C., Gehling, W. Y. Y., and Bica, A. V. D., 2006, "The Use of Filter-Paper and Suction-Plate Methods for Determining the Soil-Water Characteristic Curve of Undisturbed Colluvium Soils," *Geotech. Test. J.*, Vol. 29, No. 5, pp. 1–7.

Fredlund, D. G., Morgenstern, N. R., and Widger, R. A., 1978, "The Shear Strength of Unsaturated Soil," *Can. Geotech. J.*, Vol. 15, pp. 313–321.

Fredlund, D. G., Vanapalli, S., Xing, A., and Pufahl, D. E., 1995, "Predicting the Shear Strength Function for Unsaturated Soils Using the Soil-Water Characteristic Curve," *Proceedings of the 1st International Conference on Unsaturated Soils*, ed. Alonso, E. E., and Delage, P., Paris September 6-8, 1995, A. A. Balkema Publishers, Rotterdam, The Netherlands, pp. 63–69.

Fredlund, D. G., and Rahardjo, H., *Soil Mechanics for Unsaturated Soils*, Wiley, New York, 1993.

Fredlund, D. G. and Xing, A., 1994, "Equations for the Soil-Water Characteristic Curve," *Can. Geotech. J.*, Vol. 31, pp. 521–532.

Fredlund, D. G., Xing, A., Fredlund, M. D., and Barbour, S. L., 1996, "The Relationship of the Unsaturated Soil Shear Strength to the Soil-Water Characteristic Curve," *Can. Geotech. J.*, Vol. 33, No. 3, pp. 440–448.

Gan, J. K. M., and Fredlund, D. G., 1996, "Shear Strength Characteristics of Two Saprolitic Soils," *Can. Geotech. J.*, Vol. 33, No. 4, pp. 595–609.

Gardner, R., 1937, "A Method of Measuring the Capillary Tension of Soil Moisture Over a Wide Moisture Range," *Soil Sci.*, Vol. 43, pp. 277–283.

Gui, M. W., and Yu, C. M., 2008, "Rate of Strength Increase of Unsaturated Lateritic Soil," *Can. Geotech. J.*, Vol. 45, No. 9, pp. 1335–1343.

Houston, S. L., Houston, W. N., and Wagner, M., 1994, "Laboratory Filter Paper Suction Measurements," *Geotech. Test. J.*, Vol. 17, No. 2, pp. 1209–1217.

Jiang, G., Wang, Z., and Qiu, J., 2000, "Suction Calibration Curve of Homemade Filter Paper," *Rock Soil Mech.*, Vol. 21, No. 1, pp. 72–75. (in Chinese) Accession number: 2000045153057.

Khalili, N., and Khabbaz, M. H., 1998, "A Unique Relationship for $\chi$ for the Determination of the Shear Strength of Unsaturated Soils," *Geotechnique*, Vol. 48, No. 5, pp. 681–687.

Kranhn, J., and Fredlund, D. G., 1972, "On Total, Matric and Osmotic Suction," *Soil Sci.*, Vol. 1, No. 4, pp. 1047–1099.

Lang, A. R. G., 1967, "Osmotic Coefficients and Water Potentials of Sodium Chloride Solutions From 0 to 40 C," *Aust. J. Chem.*, Vol. 20, pp. 2017–2023.

Leong, E. C., He, L., and Rahardjo, H., 2002, "Factors Affecting the Filter Paper Method for Total and Matric Suction Measurements," *Geotech.Test. J.*, Vol. 25, No. 3, pp. 322–333.

Ling, H., and Yin, Z. Z., 2007, "Variation of Unsaturated Soil Strength With Water Contents," *Chin. J. Rock Mech. Eng.*, Vol. 26, No. 7, pp. 1499–1503. (in Chinese), Accession number: 20073210754207.

Marinho, F. A. M., and Oliveira, O. M., 2006, "The Filter Paper Method Revisited," *Geotech. Test. J.*, Vol. 29, No. 3, pp. 250–258.

Matsuoshi, Y., and Matsukura, Y., 2006, "Cohesion of Unsaturated Residual Soils as a Function of Volumetric Water Content," *Bull. Eng. Geol. Environ.*, Vol. 65, No. 4, pp. 449–455.

Miao, L. C., Zhong, X. C., and Yin, Z. Z., 1999, "Relationship Between Strength and Water Content of Expansive Soil," *Rock Soil Mech.*, Vol. 20, No. 2, pp. 71–75. (in Chinese), Accession number: 2000044938222.

Nam, S., Gutierrez, M., Diplas, P., Petrie, J., Wayllace, A., Lu, N., and Munoz, J. J., 2009, "Comparison of Testing Techniques and Models for Establishing the SWCC of Riverbank Soils," *Eng. Geol.*, Vol. 110, No. 1–2, pp. 1–10.

Ng, C. W. W., Cui, Y. J., Chen, R., and Delage, P., 2007, "The Axistranslation and Osmotic Techniques in Shear Testing of Unsaturated Soils: A Comparison," *Soils Found.*, Vol. 47, No. 4, pp. 674–684.

Oloo, S. Y., and Fredlund, D. G., 1996, "A Method for Determination of $\Phi^b$ for Statically Compacted Soils," *Can. Geotech. J.*, Vol. 33, pp. 272–280.

Patrick, P. K., Olsen, H. W., and Higgins, J. D., 2007, "Comparison of Chilled-Mirror Measurements and Filter Paper Estimates of Total Soil Suction," *Geotech. Test. J.*, Vol. 30, No. 5, pp. 360–367.

SL237-1999, *Specification of Soil Test*, China Water Press, Beijing, 1999. (in Chinese).

Sibley, J. W., and Williams, D. J., 1990, "A New Filter Material for Measuring Soil Suction," *Geotech. Test. J.*, Vol. 13, No. 4, pp. 381–384.

Terzaghi, K., 1936, "The Shear Resistance of Saturated Soils and Angle Between the Planes of Shear," *Proceedings, 1st International Conference on Soil Mechanics and Foundation Engineering*, Harvard University, Cambridge, MA, Vol. 1, pp. 54–56.

Tripathy, S., and Subba Rao, K. S., 2009, "Cyclic Swell-Shrink Behaviour of a Compacted Expansive Soil," *Geotech. Geol. Eng. J.*, Vol. 27, pp. 89–103.

van der Raat, P., Fredlund, D. G., Clifton, A. W., Klassen, M. J. K., and Jubien, W. E., 1987, "Soil Suction Measurements at Several Sites in Western Canada," *Transp. Res. Rec.*, No. 1137, pp. 24–35. Accession Number: 00470963, ISSN: 0361-1981.

Wang, Z., Yang, J. X., Kuang, J. J., An, J. Y., and Luo, Y. D., 2003, "Application of Filter Paper Method in Field Measurement of Matrix Suction," *Chin. J. Geotech. Eng.*, Vol. 25, No. 4, pp. 405–408. (in Chinese), Accession number: 2003377629378.